

Objective:

- To analyze transformations of functions

Common Core Content Standard:

- F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + a$, $af(x)$, $f(ax)$, and $f(x + a)$ for the specific values of a (both positive and negative) find the value of a given the graphs.

Shifts, stretches, ^{compressions}shrinks, and reflections are called transformations. many graphs of functions can be created from a combination of these.

Vertical and Horizontal Shifts

Let a be a positive real number. Vertical and horizontal shifts in the graph of $y = f(x)$ are represented as follows.

- Vertical shift a units UP: $\rightarrow f(x) = x^2 + 2$ UP 2 units
 $h(x) = f(x) + a$
- Vertical shift a units DOWN: $\rightarrow f(x) = x^2 - 2$ DOWN 2 units
 $h(x) = f(x) - a$
- Horizontal shift a units to the RIGHT: $f(x) = (x - 2)^2$ Right 2 units
 $h(x) = f(x - a)$
- Horizontal shift a units to the LEFT: $f(x) = (x + 2)^2$ Left 2.
 $h(x) = f(x + a)$

Reflections

- Reflection in the x-axis: Make the whole function negative
 $h(x) = -f(x)$
Original: $f(x) = 2 - x^2$ Reflected: $-(2 - x^2)$
- Reflection in the y-axis: Replace x with $-x$.
 $h(x) = f(-x)$
Original: $f(x) = 4x + 1$ Reflected: $4(-x) + 1$

Stretches and Compressions

- Vertical stretch (if $a > 1$) $f(x) = 3x^2$ $3 > 1$ stretch
 $h(x) = af(x)$
- Vertical ^{compression}shrink (if $0 < a < 1$) $f(x) = \frac{1}{3}x^2$ $0 < \frac{1}{3} < 1$ compression
 $h(x) = af(x)$
- ~~Horizontal shrink (if $a > 1$)
 $h(x) = f(ax)$~~
- ~~Horizontal stretch (if $0 < a < 1$)
 $h(x) = f(ax)$~~